NON-COMMUTATIVE CREPANT RESOLUTIONS VIA TILTING BUNDLES

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The concept of non-commutative crepant resolutions (NCCRs) was introduced by Van den Bergh [4, 5] as an algebraic counterpart of crepant resolutions of singularities. They are defined by a certain distinguished properties of (non-commutative) algebras, called *Calabi-Yau algebras*. Via the Auslander correspondence [2], the NCCRs for certain commutative algebras are nothing but endomorphism rings of cluster tilting objects, which has lead to fruitful connections between algebraic geometry and reprsentation theory.

We present a construction of NCCRs for some commutative Gorenstein rings by means of tilting bundles over some projective varieties. Let X be a smooth projective variety, and let E be a vector bundle on X whose determinant is ω_X, so that the total space Y of E is a (non-proper) Calabi-Yau variety. The fundamental case $E = \omega_X$ has played a crucial role since Van den Bergh's first examples of NCCRs [4]. Also, the operation of taking the total space of ω_X is algebraically formulated as the Calabi-Yau completion [3] of differential graded (dg) algebras which is a universal way of constructing Calabi-Yau (dg) algebras. In this talk we investigate its higher rank analgoue, that is, when E has rank possibly greater than 1. We also relate its construction to a 'higher rank' analogue of the Calabi-Yau completion [1].

This is based on a joint work with Yusuke Nakajima.

References

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